Binary brown dwarfs in the Galactic halo?

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ABSTRACT

Microlensing events towards the Large Magellanic Cloud imply that a sizeable fraction of dark matter is in the form of MACHOs (Massive Astrophysical Compact Halo Objects), presumably located in the halo of the Galaxy. Within the present uncertainties, brown dwarfs are viable candidates for MACHOs. Various considerations strongly suggest that a large number of MACHOs should actually consist of binary brown dwarfs. Yet this circumstance appears to be in flat contradiction with the fact that MACHOs have been detected as unresolved objects so far. We show that such an apparent paradox does not exist within a model in which MACHOs are clumped into dark clusters along with cold molecular clouds, since dynamical friction on these clouds makes binary brown dwarfs very close. Moreover, we argue that future microlensing experiments with more accurate photometric observations can resolve binary brown dwarfs.

Key words: stars: low-mass, brown dwarfs – Galaxy: halo – dark matter – gravitational lensing.

1 INTRODUCTION

As is well known (Trimble 1987; Carr 1994), the flat rotation curves in spiral galaxies supply the fundamental information that a large amount of dark matter is concealed in galactic haloes, but unfortunately fail to provide any hint as to its constituents.

As first proposed by Paczyński (1986, 1996), gravitational microlensing can greatly help to clarify the nature of dark matter, and since 1993 this dream has started to become a reality with the detection of several microlensing events towards the Large Magellanic Cloud (LMC) (Alcock et al. 1993; Aubourg et al. 1993).

Today, although the evidence for Massive Astrophysical Compact Halo Objects (MACHOs) – presumably located in the halo of our Galaxy – is firm, the implications of such a discovery crucially depend on the assumed Galactic model (Gates, Gyuk & Turner 1996).[1] What can be reliably concluded from the existing data set is that MACHOs should lie in the mass range 0.05–1.0 $M_\odot$, but stronger claims are unwarranted because the MACHO mass depends strongly on the uncertain properties of the considered Galactic model (De Paolis, Ingrosso & Jetzer 1996; Evans 1996).

High mass values ($\gtrsim 0.5\,M_\odot$) suggest white dwarfs as a natural candidate for MACHOs (Tamanaha et al. 1990; Adams & Laughlin 1996; Chabrier, Segretein & Méra 1996; Fields, Mathews & Schramm 1997). Besides requiring a rather ad hoc initial mass function sharply peaked somewhere in the range 1–8 $M_\odot$ and a halo age greater than ~ 16 Gyr, this option implies an unobserved increased metallicity of the interstellar medium (Gibson & Mould 1997).

Somewhat lower mass values ($\sim 0.3\,M_\odot$) point towards the possibility that MACHOs are hydrogen-burning, main-sequence M-dwarfs. However, the null results of several searches for low-mass stars in our Galaxy (Hu et al. 1994) imply that the halo cannot be mostly in the form of M-

[1]It has become customary to take the standard spherical halo model as a baseline for comparison. Unfortunately, because of the presently available limited statistics, different data-analysis procedures lead to results which are only marginally consistent. Indeed, within the standard halo model, the average MACHO mass value reported by the MACHO team is $0.5^{+0.3}_{-0.2}\,M_\odot$ (Alcock et al. 1997), whereas the mass moment method (De Rújula, Jetzer & Massó 1991) yields 0.27 $M_\odot$ (Jetzer 1996).

dwarfs, and the same conclusion is reached by optical imaging of high-latitude fields taken with the Wide Field Camera of the *Hubble Space Telescope* (*HST*) (Bahcall et al. 1994). Observe that these results are derived under the assumption of a smooth spatial distribution of M-dwarfs, and become considerably less severe in the case of a clumpy distribution (Kerins 1997a,b).

Still lower mass values ($\lesssim 0.1\ \mathrm{M}_\odot$) make brown dwarfs an attractive candidate for MACHOs.[2] Actually, precisely these mass values are supported by non-standard halo models – like the maximal disc model (van Albada & Sancisi 1986; Persic & Salucci 1990) – which tend to be favoured by recent observational data (Sackett 1997).

In spite of the fact that present uncertainties do not permit us to make any definitive statement about the nature of MACHOs, brown dwarfs nevertheless look like a viable possibility to date, and we shall maintain this assumption throughout.

Once this idea is accepted, a few almost obvious consequences follow. Given the fact that ordinary halo stars form in (globular) clusters, it seems more likely that brown dwarfs also form in clusters (to be referred to as *dark clusters*), rather than in isolation. Furthermore, since no known star formation mechanism is very efficient, we expect a substantial fraction of the primordial gas to be left over. Finally, because brown dwarfs do not give rise to stellar winds, this gas should presumably remain confined within the dark clusters.[3] As a matter of fact, it has been shown (De Paolis et al. 1995a,b; Gerhard & Silk 1996) that these suggestions can be turned into a consistent scenario, which encompasses the Fall & Rees theory for the formation of globular clusters (Fall & Rees 1985), and indeed predicts that dark clusters of brown dwarfs and cold molecular clouds – mainly of H_2 – should form in the halo at galactocentric distances larger than 10–20 kpc. A slightly different model based on the presence of a strong cooling-flow phase during the formation of our Galaxy has been considered by Fabian & Nulsen (1994) and Nulsen & Fabian (1997), and leads to a halo made of low-mass objects.

Before proceeding further, we would like to note that the most promising way to test whether MACHOs are indeed clumped into dark clusters is through correlation effects in microlensing observations. For the more massive dark clusters in the range 10^5–$10^6\ \mathrm{M}_\odot$, the implied degeneracy in the spatial and velocity distributions would result in a strong autocorrelation in the sky position of microlensing events on an angular scale of 20 arcsec, along with a correlation in their duration. It has already been pointed out that fairly poor statistics would be sufficient to rule out the possibility that MACHOs are clumped into such massive clusters, while to confirm it more events are needed (Maoz 1994; Metcalf & Silk 1996). On the other hand, if either the mass of the dark clusters is considerably smaller than $10^6\ \mathrm{M}_\odot$ or the fraction of dark matter in the form of MACHOs is small, then a much larger sample of events may be required.

The aim of the present paper is to address a further issue that the above picture implies. As we shall discuss below, a large fraction of MACHOs are expected to consist of binary brown dwarfs. However, this circumstance is an occasion for anxiety, since the overwhelming majority of MACHOs look like unresolved objects in current microlensing experiments. We will demonstrate that such an apparent paradox does not actually exist within the model in question, since dynamical friction on molecular clouds makes binary brown dwarfs so close that they cannot be resolved as yet – still, we argue that they may be resolved in future microlensing experiments with more accurate photometric observations.

2 PRELIMINARY CONSIDERATIONS

A thorough account of the scenario under consideration has been reported elsewhere, along with an analysis of several observational tests (De Paolis et al. 1996). We should stress that the lack of observational information about dark clusters would make any effort to understand their structure and dynamics hopeless, were it not for some remarkable insights that our unified treatment of globular and dark clusters provides us.

In the first place, it looks quite natural to assume that dark clusters have a denser core surrounded by an extended spherical halo. Moreover, it seems reasonable to suppose (at least tentatively) that dark clusters have the same average mass density as globular clusters. Accordingly, we get $r_{\rm DC} \simeq 0.12\ (M_{\rm DC}/\mathrm{M}_\odot)^{1/3}$ pc, with $M_{\rm DC}$ and $r_{\rm DC}$ denoting the mass and median radius of a dark cluster, respectively. So, by the virial theorem the one-dimensional velocity dispersion σ of MACHOs and molecular clouds within a dark cluster reads

$$\sigma \simeq 6.9 \times 10^{-2} \left(\frac{M_{\rm DC}}{\mathrm{M}_\odot}\right)^{1/3} \ \mathrm{km\ s^{-1}}. \tag{1}$$

In addition, dark clusters – just like globular clusters – presumably stay for a long time in a quasi-stationary phase, with an average central density $\rho(0)$ slightly lower than $10^4\ \mathrm{M}_\odot\ \mathrm{pc}^{-3}$ (which is the observed average central density for globular clusters).

We now recall some basic points of the considered picture, which are important for the considerations to follow. First, the amount of virialized diffuse gas inside dark clusters should be small – otherwise it should have been observed in the radio band – and so most of the gas has to be in the form of cold self-gravitating clouds clumped into dark clusters. Secondly, at variance with the case of globular clusters, the mass distribution of dark clusters should be smooth, with a corresponding spectrum extending from $M_{\rm DC} \sim 10^6\ \mathrm{M}_\odot$ down to much lower values. Thirdly, the dark clusters with $3 \times 10^2 \lesssim M_{\rm DC} \lesssim 10^6\ \mathrm{M}_\odot$ are expected to have survived all disruptive effects and should still populate the outer Galactic halo today. Fourthly, the dark clusters with $M_{\rm DC} \lesssim 5 \times 10^4\ \mathrm{M}_\odot$ should have begun core collapse. Fifthly, since the temperature of these clouds is close to that of the cosmic background radiation (De Paolis et al. 1995c), the

[2] We stress that the limit for hydrogen burning – usually quoted as $0.08\ \mathrm{M}_\odot$ – getes increased up to $0.11\ \mathrm{M}_\odot$ for low-metallicity objects, such as a halo population (D'Antona 1987; Burrows, Hubbard & Lunine 1989).

[3] We emphasize that a halo primarily made of *unclustered* brown dwarfs (as well as white and M-dwarfs) would contain too much diffuse hot gas (at virial temperature $\sim 10^6$ K) which emits in the X-ray band.

virial theorem implies

$$r_{\rm m} \simeq 4.8 \times 10^{-2} \left(\frac{M_{\rm m}}{\rm M_{\odot}}\right) \text{pc}, \tag{2}$$

where $r_{\rm m}$ and $M_{\rm m}$ indicate the median radius and mass, respectively, of a self-gravitating cloud. Accordingly, the average number density $n_{\rm m}$ in a cloud turns out to be

$$n_{\rm m} \simeq 62.2 \left(\frac{\text{pc}}{r_{\rm m}}\right)^2 \text{cm}^{-3}. \tag{3}$$

Finally, the leftover gas is mainly H_2.

We suppose for definiteness (and with an eye to microlensing experiments) that all individual brown dwarfs have mass $m \simeq 0.1\ \rm M_{\odot}$. Furthermore, since molecular clouds also originate from the fragmentation process which produces the brown dwarfs, we suppose (for definiteness) that they lie in the mass range $10^{-3} \lesssim M_{\rm m} \lesssim 10^{-1}\ \rm M_{\odot}$. Correspondingly, equations (2) and (3) yield $4.8 \times 10^{-5} \lesssim r_{\rm m} \lesssim 4.8 \times 10^{-3}$ pc and $2.7 \times 10^{10} \gtrsim n_{\rm m} \gtrsim 2.7 \times 10^6\ \text{cm}^{-3}$, respectively.

3 MACHOS AS BINARY BROWN DWARFS

In much the same way as it occurs for ordinary stars, the fragmentation mechanism that gives rise to individual brown dwarfs should, in the present case, presumably produce a substantial fraction of binary brown dwarfs – these will be referred to as *primordial* binaries. It is important to keep in mind that their mass fraction $f_{\rm PB}$ can be as large as 50 per cent, and that – because of the mass stratification instability – they will be concentrated inside the dark cluster cores (Spitzer 1987), which are therefore expected to be chiefly composed of binaries and molecular clouds. In addition, as far as dark clusters with $M_{\rm DC} \lesssim 5 \times 10^4\ \rm M_{\odot}$ are concerned, *tidally captured* binary brown dwarfs ought to form in the cores because of the increased central density caused by core collapse (Fabian, Pringle & Rees 1975). Thus we are led to the conclusion that MACHOs should consist of both *individual* and *binary* brown dwarfs within the present picture.

Our subsequent analysis will be carried out in the general case of arbitrary $f_{\rm PB}$, but our interest is, of course, focused on the case of large $f_{\rm PB}$.

We recall that a binary system is hard when its binding energy exceeds the kinetic energy of field stars (otherwise it is soft). As is well known, soft binaries always get softer, whereas hard binaries always get harder because of encounters with individual stars (Heggie 1975). In the case under consideration, binary brown dwarfs happen to be hard when their orbital radius a obeys the constraint

$$a \lesssim 1.4 \times 10^{12} \left(\frac{\rm M_{\odot}}{M_{\rm DC}}\right)^{2/3} \text{km}. \tag{4}$$

Hence only those binaries which satisfy condition (4) can survive up until the present.

Now, consistency with the results of microlensing experiments – in the sense specified above – requires that the present orbital radius of the overwhelming majority of binary brown dwarfs should be (roughly) less than one-half of the Einstein radius for microlensing towards the LMC (Gaudi & Gould 1997). This demand implies in turn that the further constraint

$$a_{\rm today} \lesssim 3 \times 10^8\ \text{km} \tag{5}$$

has to be met in the present. What is crucial to realize is that the latter bound turns out to be *stronger* than the former for $M_{\rm DC} \lesssim 3.2 \times 10^5\ \rm M_{\odot}$.

Let us consider first *tidally captured* binaries, the formation of which is ultimately brought about by core collapse in dark clusters, and is expected to proceed as in the case of globular clusters. So, following the analysis of Press & Teukolsky (1977) and Lee & Ostriker (1986), we deduce that practically all individual brown dwarfs in a dark cluster core get captured into binaries as soon as the core density[4] $\rho(0)$ starts to satisfy the condition

$$\rho(0) \gtrsim \frac{3.2 \times 10^4}{f_{\rm IBD}} \left(\frac{M_{\rm DC}}{\rm M_{\odot}}\right)^{0.4} \rm M_{\odot}\ pc^{-3}, \tag{6}$$

where $f_{\rm IBD}$ denotes the mass fraction of individual brown dwarfs in the core. According to the above assumptions, we expect $\rho(0) \simeq 10^4\ \rm M_{\odot}\ pc^{-3}$ *just before core collapse*, and so a moderate increase in $\rho(0)$ may be sufficient to make tidal capture operative – hence this process would occur at the onset of core collapse. However, this conclusion depends on $f_{\rm PB}$, since $f_{\rm IBD}$ necessarily becomes small for large $f_{\rm PB}$. What is the orbital radius of tidally captured binary brown dwarfs? Following the procedure outlined by Statler, Ostriker & Cohn (1987), we easily find $a \simeq 2.5 \times 10^5$ km (this value is almost independent of $M_{\rm DC}$), and so tidally captured binaries are very hard and obviously obey condition (5). Unfortunately, tidally captured binary dwarfs are irrelevant from the observational point of view, since their fractional abundance turns out to be less than 1 per cent in any case.

What is the fate of *primordial* binaries? Surely only those which are hard can survive. In fact, were individual–binary encounters the only relevant process, we would conclude that hard binary brown dwarfs should indeed survive. However, binary–binary encounters also play an important role in the dark cluster cores, where binaries are far more abundant than individual brown dwarfs as long as $f_{\rm PB}$ is not negligibly small. Now, it is well known that in the latter process one of two binaries often gets disrupted (this cannot happen for both binaries – given that they are hard – while fly-bys are rather infrequent), thereby leading to the depletion of the primordial binary population. We will discuss this effect in Section 5, where we shall find that, for realistic values of the dark cluster parameters, the binary break-up does not take place. Still, this is not the end of the story, because all values for their orbital radius consistent with condition (4) are in principle allowed. Thus it is crucial that primordial binaries manage to shrink in such a way that *condition (5) is also eventually met.*

4 HARDENING PROCESSES

Let us turn our attention to the specific mechanism by which condition (5) can become fulfilled. Observe that this

[4]We suppose throughout that the core of dark clusters has a constant density profile.

requirement demands that a binary should give up a binding energy larger than 4.5×10^{43} erg.

4.1 Collisional hardening

Collisional hardening – namely the process whereby hard binaries get harder in encounters with individual brown dwarfs – looks like the most natural possibility. In order to see whether it works, we consider the associated average hardening rate (Spitzer & Mathieu 1980), which reads presently

$$\langle \dot{E}\rangle_{\rm CH} \simeq -2.8\,\frac{G^2 m^3 n_{\rm IBD}(0)}{\sigma}, \tag{7}$$

where $n_{\rm IBD}(0)\simeq f_{\rm IBD}\,\rho(0)/m$ is the number density of individual brown dwarfs in the core. So, equation (7) becomes

$$\langle \dot{E}\rangle_{\rm CH} \simeq -1.7\times10^{32} f_{\rm IBD}\left(\frac{\rm M_\odot}{M_{\rm DC}}\right)^{1/3} \times\left(\frac{\rho(0)}{\rm M_\odot\ pc^{-3}}\right)\ {\rm erg\ yr^{-1}}, \tag{8}$$

thanks to equation (1). From equation (8) we can estimate the total amount of binding energy $\Delta E_{\rm B}$ released by a primordial binary via encounters with individual brown dwarfs during the lifetime of the Universe.[5] Taking $\rho(0)\sim10^3\ \rm M_\odot$ $\rm pc^{-3}$ – in agreement with our assumptions – we find $\Delta E_{\rm B}\sim10^{43}$–$10^{44}\ f_{\rm IBD}$ erg (depending on $M_{\rm DC}$). As already pointed out, a large $f_{\rm PB}$ would imply a small $f_{\rm IBD}$, in which case collisional hardening fails to reduce the present orbital radius of primordial binary brown dwarfs down to a value consistent with condition (5). Furthermore, denoting by Δa the change of orbital radius corresponding to a loss of binding energy $\Delta E_{\rm B}$, we have $|\Delta E_{\rm B}/E_{\rm B}|=|\Delta a/a|$, which shows more generally that collisional hardening turns out to be irrelevant for microlensing observations.

4.2 Frictional hardening

As we shall show below, the presence of molecular clouds in the dark cluster cores – which is indeed the most characteristic feature of the model in question – provides a novel mechanism by which primordial binary brown dwarfs give up a large amount of binding energy, thereby becoming so hard that condition (5) is actually obeyed. Basically, this occurs because of dynamical friction on molecular clouds, and so it will be referred to as *frictional hardening*.

It is not difficult to extend the standard treatment of dynamical friction (Binney & Tremaine 1987) to the relative motion of the brown dwarfs in a binary system which moves inside a molecular cloud. For simplicity, we assume that molecular clouds have a constant density profile $\rho_{\rm m}$. In the case of a circular orbit,[6] the equations of motion imply that the time t_{21} needed to reduce the orbital radius a from a_1 down to a_2 is

$$t_{21}\simeq0.17\left(\frac{m}{G}\right)^{1/2}\frac{1}{\rho_{\rm m}\ \ln\Lambda}\,(a_2^{-3/2}-a_1^{-3/2}), \tag{9}$$

where the Coulomb logarithm reads $\ln\Lambda\simeq\ln(r_{\rm m}v_{\rm c}^2/Gm)\simeq\ln(r_{\rm m}/a_1)$, with $v_{\rm c}$ denoting the circular velocity (approximately given by Kepler's third law). Manifestly, the diffusion approximation – upon which the present treatment is based – requires that the orbital radius of a binary should always be smaller than the median radius of a cloud. As we are concerned henceforth with hard binaries, a_1 has to obey condition (4). On the other hand, a_1 is the larger value for the orbital radius in equation (9). So, we shall take for definiteness – in the Coulomb logarithm only – $a_1\simeq1.4\times10^{12}\,({\rm M_\odot}/M_{\rm DC})^{2/3}$ km. In addition, from equation (2) we have $\rho_{\rm m}\simeq2.5\,({\rm pc}/r_{\rm m})^2\ {\rm M_\odot\ pc^{-3}}$. Hence, putting everything together, we obtain

$$t_{21}\simeq5\times10^{25}\,\Xi\left(\frac{r_{\rm m}}{\rm pc}\right)^2\left[\left(\frac{\rm km}{a_2}\right)^{3/2}-\left(\frac{\rm km}{a_1}\right)^{3/2}\right]\ {\rm yr}, \tag{10}$$

having set

$$\Xi\equiv[3+\ln(r_{\rm m}/{\rm pc})+0.71\,\ln(M_{\rm DC}/{\rm M_\odot})]^{-1}. \tag{11}$$

Specifically, the diffusion approximation demands $\Xi>0$, which in turn yields

$$r_{\rm m}>5\times10^{-2}\left(\frac{\rm M_\odot}{M_{\rm DC}}\right)^{0.7}\ {\rm pc}. \tag{12}$$

Observe that for $M_{\rm DC}\lesssim2.1\times10^4\ \rm M_\odot$ this constraint restricts the range of allowed values of $r_{\rm m}$, as stated in Section 2.

Let us begin by estimating the time $t_{\rm frict}$ required by dynamical friction to reduce the orbital radius from $a_{\rm in}\simeq1.4\times10^{12}\ ({\rm M_\odot}/M_{\rm DC})^{2/3}$ km down to $a_{\rm fin}\simeq3\times10^8$ km, under the assumption that a binary moves all the time inside molecular clouds. Clearly, from equation (10) we get

$$t_{\rm frict}\simeq9.5\times10^{12}\,\Xi\left(\frac{r_{\rm m}}{\rm pc}\right)^2 A\ {\rm yr}, \tag{13}$$

where we have set

$$A\equiv\left[1-3.1\times10^{-6}\left(\frac{M_{\rm DC}}{\rm M_\odot}\right)\right]. \tag{14}$$

As an indication, we notice that for $r_{\rm m}\simeq10^{-3}$ pc ($M_{\rm m}\simeq2\times10^{-2}\ \rm M_\odot$) and $M_{\rm DC}\simeq10^5\ \rm M_\odot$ we find $\Xi\simeq0.2$, and so $t_{\rm frict}\simeq1.3\times10^6$ yr.

Were the dark clusters completely filled by clouds, equation (13) would be the final result. However, the distribution of the clouds is lumpy. $t_{\rm frict}$ has therefore to be understood as the total time spent by a binary inside the clouds. Thus the total time demanded to reduce $a_{\rm in}$ down to $a_{\rm fin}$ is evidently longer than $t_{\rm frict}$, and can be computed by the following procedure. Keeping in mind that both the clouds and the binaries have average velocity $v\simeq\sqrt{3}\sigma$ (for simplicity, we neglect the equipartition of kinetic energy of the binaries), it follows that the time needed by a binary to cross a single cloud is

$$t_{\rm m}\simeq\frac{r_{\rm m}}{\sqrt{2}v}\simeq5.6\times10^6\left(\frac{r_{\rm m}}{\rm pc}\right)\left(\frac{\rm M_\odot}{M_{\rm DC}}\right)^{1/3}\ {\rm yr}. \tag{15}$$

[5]The age of the Universe is nominally taken to be 10^{10} yr throughout.
[6]Indeed, the circularization of the orbit is achieved by tidal effects after a few periastron passages (Zahn 1987).

We see that the above values of $r_{\rm m}$ and $M_{\rm DC}$ imply $t_{\rm m} \simeq 1.2 \times 10^2$ yr. Therefore frictional hardening involves many clouds and is actually accomplished only after

$$N_{\rm m} \simeq \frac{t_{\rm frict}}{t_{\rm m}} \simeq 1.7 \times 10^6\, \Xi \left(\frac{r_{\rm m}}{\rm pc}\right)\left(\frac{M_{\rm DC}}{\rm M_\odot}\right)^{1/3} A \tag{16}$$

clouds have been traversed. Taking again the above values for $r_{\rm m}$ and $M_{\rm DC}$, we get $N_{\rm m} \simeq 1.1 \times 10^4$.

It is illuminating to find how many crossings of the core are necessary for a binary to traverse $N_{\rm m}$ clouds. To this end, we proceed to estimate the number of clouds $N_{\rm c}$ encountered during *one* crossing of the core. Describing the dark clusters by a King model, we can identify the core radius with the King radius. Evidently, the cross-section for binary–cloud encounters is $\pi r_{\rm m}^2$, and so we have

$$N_{\rm c} \simeq \left[\frac{9\sigma^2}{4\pi G\rho(0)}\right]^{1/2} n_{\rm CL}(0)\, \pi r_{\rm m}^2, \tag{17}$$

with $n_{\rm CL}(0)$ denoting the cloud number density in the core. Thanks to equation (2), we can write

$$n_{\rm CL}(0) \simeq f_{\rm CL} \frac{\rho(0)}{M_{\rm m}}$$

$$\simeq 4.8 \times 10^{-2} f_{\rm CL} \left(\frac{\rm pc}{r_{\rm m}}\right)\left[\frac{\rho(0)}{\rm M_\odot\ pc^{-3}}\right] {\rm pc}^{-3}, \tag{18}$$

where $f_{\rm CL}$ denotes the fraction of core dark matter in the form of molecular clouds. Correspondingly, equation (17) becomes

$$N_{\rm c} \simeq 0.13 \times f_{\rm CL} \left[\frac{\rho(0)}{\rm M_\odot\ pc^{-3}}\right]^{1/2} \left(\frac{M_{\rm DC}}{\rm M_\odot}\right)^{1/3} \left(\frac{r_{\rm m}}{\rm pc}\right), \tag{19}$$

on account of equation (1). So the total number of core crossings $N_{\rm cc}$ that a binary has to make in order to reduce its orbital radius down to $a_{\rm fin} \simeq 3 \times 10^8$ km is

$$N_{\rm cc} \simeq \frac{N_{\rm m}}{N_{\rm c}} \simeq 1.3 \times 10^7 f_{\rm CL}^{-1}\, \Xi \left[\frac{\rm M_\odot\ pc^{-3}}{\rho(0)}\right]^{1/2} A. \tag{20}$$

On the other hand, the core crossing time is

$$t_{\rm cc} \simeq \left[\frac{9\sigma^2}{4\pi G\rho(0)}\right]^{1/2} \frac{1}{v}$$

$$\simeq 7 \times 10^6 \left[\frac{\rm M_\odot\ pc^{-3}}{\rho(0)}\right]^{1/2} {\rm yr}. \tag{21}$$

Thus the total time needed by frictional hardening to make primordial binary brown dwarfs unresolvable in present microlensing experiments is

$$t_{\rm tot} \simeq N_{\rm cc} t_{\rm cc} \simeq 9 \times 10^{13} f_{\rm CL}^{-1}\, \Xi \left[\frac{\rm M_\odot\ pc^{-3}}{\rho(0)}\right] A\ {\rm yr}. \tag{22}$$

Observe that $t_{\rm tot}$ is insensitive to $r_{\rm m}$ and almost insensitive to $M_{\rm DC}$ (this dependence enters only logarithmically in Ξ). As a matter of fact, since we have considered so far only the extreme case of largest $a_{\rm m}$, $t_{\rm tot}$ in equation (22) should be understood as an *upper bound* to the total hardening time. Accordingly, taking $f_{\rm CL} \simeq 0.5$, we see that for the above illustrative values of $r_{\rm m}$ and $M_{\rm DC}$ we get $t_{\rm tot}$ shorter than the age of the Universe for $\rho(0) \gtrsim 2.5 \times 10^3\ {\rm M_\odot\ pc^{-3}}$. As the latter value is consistent with our assumptions, we conclude that there is enough time for frictional hardening to operate.

5 BINARY–BINARY ENCOUNTERS

We are now in a position to investigate the implications of binary–binary encounters.

As a first step, we recall that the average rate Γ for individual–binary (IB) and binary–binary (BB) encounters (Spitzer & Mathieu 1980) can presently be written as

$$\Gamma \simeq \alpha \frac{Gma}{\sigma}, \tag{23}$$

with α being either $\alpha_{\rm IB} \simeq 14.3$ or $\alpha_{\rm BB} \simeq 13$. Correspondingly, on account of equation (1) the reaction time in the dark cluster cores turns out to be

$$t_{\rm react} \simeq 10^{19} \beta \left[\frac{\rm M_\odot\ pc^{-3}}{\rho(0)}\right]\left(\frac{M_{\rm DC}}{\rm M_\odot}\right)^{1/3} \left(\frac{\rm km}{a}\right) {\rm yr}, \tag{24}$$

with β being either $\beta_{\rm IB} \simeq 3.1/f_{\rm IBD}$ or $\beta_{\rm BB} \simeq 6.7/f_{\rm PB}$. Since both processes are a priori of comparable strength and in the cores we expect $f_{\rm IBD} \ll f_{\rm PB}$, individual–binary encounters will be neglected.

As already stressed, binary–binary encounters can lead to their disruption. Now, if no hardening were to occur – which means that the orbital radius a would stay constant – the binary survival condition would simply follow by demanding that $t_{\rm react}^{\rm BB}$ should exceed the age of the Universe. However, hardening makes a decrease, and so $t_{\rm react}^{\rm BB}$ increases with time. This effect can be taken into account by considering the average value $\langle t_{\rm react}^{\rm BB} \rangle$ of the reaction time over the time interval in question (to be denoted by T), namely

$$\langle t_{\rm react}^{\rm BB} \rangle \equiv \frac{1}{T} \int_0^T {\rm d}t\, t_{\rm react}^{\rm BB}. \tag{25}$$

In order to compute $\langle t_{\rm react}^{\rm BB} \rangle$, the temporal dependence $a(t)$ of the binary orbital radius is required. This quantity can be obtained as follows. We have seen that the total time $t_{21}^{\rm TOT}$ needed to reduce a_1 down to a_2 is actually longer than t_{21}, since the cloud distribution is lumpy. So, by going through the same steps as before, we easily find

$$t_{21}^{\rm TOT} \simeq 4.8 \times 10^{26}\, \Xi\, f_{\rm CL}^{-1} \left(\frac{\rm M_\odot\ pc^{-3}}{\rho(0)}\right)\left[\left(\frac{\rm km}{a_2}\right)^{3/2} - \left(\frac{\rm km}{a_1}\right)^{3/2}\right] {\rm yr}. \tag{26}$$

Setting, for notational convenience, $t \equiv t_{21}^{\rm TOT}$, $a_0 \equiv a_1$ and $a(t) \equiv a_2$, equation (26) yields

$$a(t) \simeq \left\{\left(\frac{\rm km}{a_0}\right)^{3/2} + 2.1 \times 10^{-27}\, \Xi^{-1} f_{\rm CL}\right.$$

$$\left. \times \left[\frac{\rho(0)}{\rm M_\odot\ pc^{-3}}\right]\left(\frac{t}{\rm yr}\right)\right\}^{-2/3} {\rm km}. \tag{27}$$

Combining equations (24) and (27) together and inserting the ensuing expression into equation (25), we get

$$\langle t^{\rm BB}_{\rm react} \rangle \simeq 1.9 \times 10^{46} f_{\rm PB}^{-1} f_{\rm CL}^{-1} \Xi \left(\frac{M_{\rm DC}}{\rm M_\odot}\right)^{1/3} \left(\frac{\rm yr}{T}\right) \times \left(\frac{\rm km}{a_0}\right)^{5/2} \left[\frac{\rm M_\odot\ pc^{-3}}{\rho(0)}\right]^2 \left[\!\!\left[\left\{ 1 + 2.1 \times 10^{-27} \Xi^{-1} f_{\rm CL} \times \left[\frac{\rho(0)}{\rm M_\odot\ pc^{-3}}\right] \left(\frac{T}{\rm yr}\right) \left(\frac{a_0}{\rm km}\right)^{3/2} \right\}^{5/3} - 1 \right]\!\!\right] \,{\rm yr}. \quad (28)$$

Let us now require $\langle t^{\rm BB}_{\rm react} \rangle$ to exceed the age of the Universe (taking evidently $T \simeq 10^{10}$ yr). As is apparent from equation (24), $t^{\rm BB}_{\rm react}$ is shorter for softer binaries. Hence, in order to contemplate *hard* binaries with an arbitrary orbital radius, we have to set $a_0 \simeq 1.4 \times 10^{12} ({\rm M_\odot}/M_{\rm DC})^{2/3}$ km in equation (28). Correspondingly, the binary survival condition reads[7]

$$f_{\rm PB} \lesssim 8.2 \times 10^{-5} f_{\rm CL}^{-1} \Xi \left(\frac{M_{\rm DC}}{\rm M_\odot}\right)^2 \left[\frac{\rm M_\odot\ pc^{-3}}{\rho(0)}\right]^2 \times \left[\!\!\left[\left\{ 1 + 35 f_{\rm CL} \Xi^{-1} \left[\frac{\rho(0)}{\rm M_\odot\ pc^{-3}}\right] \left(\frac{\rm M_\odot}{M_{\rm DC}}\right) \right\}^{5/3} - 1 \right]\!\!\right]. \quad (29)$$

Although the presence of various dark cluster parameters prevents a clear-cut conclusion to be drawn from condition (29), in the illustrative case $M_{\rm DC} \simeq 10^5\,{\rm M_\odot}$ and $f_{\rm CL} \simeq 0.5$ condition (29) imply, e.g., $f_{\rm PB} \lesssim 0.3$ for $\rho(0) \simeq 3 \times 10^3\,{\rm M_\odot\ pc^{-3}}$. Thus we infer that for realistic values of the parameters in question, a sizeable fraction of primordial binary brown dwarfs survive binary–binary encounters in the core.[8]

Binary–binary encounters also have an implication of direct relevance for frictional hardening. Indeed, before claiming that such a mechanism really accomplishes its job, we have to make sure that primordial binaries do not leave the core before their orbital radius is reduced down to acceptable values. As in the case of globular clusters, encounters between individual and binary (IB) brown dwarfs can give them enough kinetic energy to escape from the core, and the same happens in encounters between binaries (BB) (Spitzer 1987). As explained above, we can restrict our attention to the latter process, and we consider $\langle t^{\rm BB}_{\rm react} \rangle$ as averaged over the total hardening time $t_{\rm tot}$ (so we are going to take $T \simeq t_{\rm tot}$ in equation 28). Again, in order to contemplate *hard* binaries with an arbitrary orbital radius, we set $a_0 \simeq 1.4 \times 10^{12} ({\rm M_\odot}/M_{\rm DC})^{2/3}$ km in equation (28). Recalling equation (22), we find

$$\frac{\langle t^{\rm BB}_{\rm react} \rangle}{t_{\rm tot}} \simeq 10^{-21} f_{\rm CL} f_{\rm PB}^{-1} \Xi^{-1} A^{-2} \left(\frac{M_{\rm DC}}{\rm M_\odot}\right) \times \left\{ \left[1 + 3.1 \times 10^5 A \left(\frac{\rm M_\odot}{M_{\rm DC}}\right) \right]^{5/3} - 1 \right\} {\rm yr}. \quad (30)$$

[7]Application of the same argument to tidally captured binaries shows that no depletion occurs in this way.

[8]If the initial value of $f_{\rm PB}$ fails to satisfy condition (29), primordial binaries start to be destroyed in binary–binary encounters until their fractional abundance is reduced down to a value consistent with condition (29).

As is the case of equation (29), the various parameters showing up in equation (30) do not permit us to make definitive statements, but in the illustrative case, $M_{\rm DC} \simeq 10^5\,{\rm M_\odot}$, $f_{\rm CL} \simeq 0.5$ and $f_{\rm PB} \lesssim 0.3$, equation (30) yields $\langle t^{\rm BB}_{\rm react} \rangle \gtrsim t_{\rm tot}$. In conclusion, it seems fair to say that condition (5) is ultimately met by the overwhelming majority of binary brown dwarfs (this statement is especially true in view of the fact that $t_{\rm tot}$ – as given by equation 22 – has to be viewed as an upper bound to the total hardening time).

6 ORBITAL RADIUS OF PRIMORDIAL BINARIES

Our concern was primarily to show that primordial binaries are hard enough today to avoid conflict with the results of microlensing experiments. Still, it is important to know how large their present orbital radius is. Unfortunately, the lack of information about the formation mechanism precludes a clear-cut answer, but it is remarkable that a non-trivial – and in fact very interesting – lower bound can nevertheless be derived. Basically, the argument is very similar to the previous one that led us to equation (26). However, it will now be used with a different logic, for we shall leave both a_1 and a_2 free while demanding that $t^{\rm TOT}_{21}$ should equal the age of the Universe. Correspondingly, equation (26) implies

$$\left(\frac{\rm km}{a_2}\right)^{3.2} \simeq \left(\frac{\rm km}{a_1}\right)^{3/2} + 2.1 \times 10^{-17} f_{\rm CL} \Xi^{-1} \left[\frac{\rho(0)}{\rm M_\odot\ pc^{-3}}\right]. \quad (31)$$

Manifestly, frictional hardening is operative to the extent that a_2 becomes considerably smaller than a_1. Accordingly, from equation (31) we see that this is indeed the case, provided that

$$a_1 \gtrsim 1.3 \times 10^{11} f_{\rm CL}^{-2/3} \Xi^{2/3} \left[\frac{\rm M_\odot\ pc^{-3}}{\rho(0)}\right]^{2/3} {\rm km}. \quad (32)$$

Owing to condition (32), equation (31) entails

$$a_1 \simeq 1.3 \times 10^{11} f_{\rm CL}^{-2/3} \Xi^{2/3} \left[\frac{\rm M_\odot\ pc^{-3}}{\rho(0)}\right]^{2/3} {\rm km}. \quad (33)$$

Physically, the emerging picture is as follows. Only those primordial binaries whose initial orbital radius satisfies condition (32) are affected by frictional hardening, and their present orbital radius turns out to be almost *independent* of the initial value. We can make the present discussion somewhat more specific by noticing that our assumptions strongly suggest $\rho(0) \lesssim 10^4\,{\rm M_\odot\ pc^{-3}}$, in which case both condition (32) and equation (33) acquire the form

$$a_{1,2} \gtrsim 2.8 \times 10^8 f_{\rm CL}^{-2/3} \Xi^{2/3}\,{\rm km}. \quad (34)$$

Evidently, very hard primordial binaries – which violate condition (34) – do not suffer frictional hardening, and the same is true for tidally captured binaries. We stress that these conclusions are (practically) unaffected by collisional hardening.

7 ENERGY BALANCE

As the above analysis shows, dynamical friction transfers a huge amount of energy from primordial binary brown dwarfs to molecular clouds, and so it is compelling to investigate (at least) the gross features of the energy balance.

Let us start by evaluating the energy acquired by molecular clouds in the process of frictional hardening. Recalling that the traversal time for a single cloud is given by equation (15), equation (10) requires that – after a binary with initial orbital radius a_1 has crossed N clouds – its orbital radius is reduced to

$$a_{N+1} \simeq \left[B_N \left(\frac{\mathrm{M}_\odot}{M_{\mathrm{DC}}} \right)^{1/3} + \left(\frac{\mathrm{km}}{a_1} \right)^{3/2} \right]^{-2/3} \mathrm{km}, \tag{35}$$

with $B_N \equiv 1.1 \times 10^{-19} N \Xi^{-1} (\mathrm{pc}/r_{\mathrm{m}})$. Accordingly, we see that the orbital radius remains almost constant until N reaches the critical value

$$N_* \equiv 9 \times 10^{18} \Xi \left(\frac{r_{\mathrm{m}}}{\mathrm{pc}} \right) \left(\frac{M_{\mathrm{DC}}}{\mathrm{M}_\odot} \right)^{1/3} \left(\frac{\mathrm{km}}{a_1} \right)^{3/2}, \tag{36}$$

whereas it *decreases* afterwards. Because the energy acquired by the clouds is just the binding energy given up by primordial binaries, the above information can be directly used to compute the energy $\Delta E_{\mathrm{c}}(N)$ gained by the Nth cloud traversed by a binary with initial orbital radius a_1. Manifestly, we have

$$\Delta E_{\mathrm{c}}(N) = \frac{1}{2} G m^2 \left(\frac{1}{a_{N+1}} - \frac{1}{a_N} \right). \tag{37}$$

Thanks to equation (35), a straightforward calculation shows that $\Delta E_{\mathrm{c}}(N)$ stays practically constant:

$$\Delta E_{\mathrm{c}} \simeq 9.8 \times 10^{32} \Xi^{-1} \left(\frac{\mathrm{pc}}{r_{\mathrm{m}}} \right) \left(\frac{\mathrm{M}_\odot}{M_{\mathrm{DC}}} \right)^{1/3} \left(\frac{a_1}{\mathrm{km}} \right)^{1/2} \mathrm{erg}, \tag{38}$$

as long as $N \lesssim N_*$, while it subsequently *decreases*. So the amount of energy transferred to a cloud is maximal during the early stages of hardening. Now, since the binding energy of a cloud is

$$E_{\mathrm{c}} \simeq 7.7 \times 10^{42} \left(\frac{r_{\mathrm{m}}}{\mathrm{pc}} \right) \mathrm{erg}, \tag{39}$$

it can well happen that $\Delta E_{\mathrm{c}} > E_{\mathrm{c}}$ (depending on r_{m}, M_{DC} and a_1), which means that the cloud would evaporate unless it manages to efficiently dispose of the excess energy.

A deeper insight into this issue can be gained as follows (we focus our attention on the early stages of hardening, when the effect under consideration is more dramatic). Imagine that a spherical cloud is crossed by a primordial binary which moves along a straight line, and consider the cylinder Δ traced by the binary inside the cloud (its volume being approximately $\pi a^2 r_{\mathrm{m}}$). Hence, by equation (3), the average number of molecules inside Δ turns out to be

$$N_\Delta \simeq 5.8 \times 10^{30} \left(\frac{a_1}{\mathrm{km}} \right)^2 \left(\frac{\mathrm{pc}}{r_{\mathrm{m}}} \right). \tag{40}$$

Physically, the energy ΔE_{c} is first deposited within Δ in the form of heat. Neglecting thermal conductivity (more about this later), the temperature inside Δ accordingly becomes

$$T_\Delta \simeq \frac{2}{3} \frac{\Delta E_{\mathrm{c}}}{N_\Delta k_{\mathrm{B}}}$$
$$\simeq 8.1 \times 10^{17} \Xi^{-1} \left(\frac{\mathrm{M}_\odot}{M_{\mathrm{DC}}} \right)^{1/3} \left(\frac{\mathrm{km}}{a_1} \right)^{3/2} \mathrm{K}, \tag{41}$$

k_{B} being the Boltzmann constant. On account of conditions (4) and (32), equation (41) yields

$$0.5 \Xi^{-1} \left(\frac{M_{\mathrm{DC}}}{\mathrm{M}_\odot} \right)^{2/3} \mathrm{K} \lesssim T_\Delta \lesssim 16.2 f_{\mathrm{CL}} \Xi^{-2} \left[\frac{\rho(0)}{\mathrm{M}} \right] \left(\frac{\mathrm{M}_\odot}{M_{\mathrm{DC}}} \right)^{1/3} \mathrm{K}, \tag{42}$$

which – in the illustrative case $f_{\mathrm{CL}} \simeq 0.5$, $\rho(0) \simeq 3 \times 10^3\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}$ and $M_{\mathrm{DC}} \simeq 10^5\,\mathrm{M}_\odot$ – implies in turn $5.3 \times 10^3 \lesssim T_\Delta \lesssim 1.3 \times 10^4\,\mathrm{K}$. As a consequence of the increased temperature, the molecules within Δ will radiate, thereby reducing the excess energy in the cloud. In order to see whether this mechanism actually prevents the cloud from evaporating, we notice that the characteristic time needed to accumulate the energy ΔE_{c} inside Δ is just the traversal time t_{m}. This energy will therefore be totally radiated away, provided that the cooling rate per molecule Λ exceeds the critical value Λ_* given by the equilibrium condition

$$N_\Delta \Lambda_* t_{\mathrm{m}} \simeq \Delta E_{\mathrm{c}}. \tag{43}$$

Specifically, equation (43) yields

$$\Lambda_* \simeq 10^{-12} \Xi^{-1} \left(\frac{\mathrm{pc}}{r_{\mathrm{m}}} \right) \left(\frac{\mathrm{km}}{a_1} \right)^{3/2} \mathrm{erg\ s^{-1}\ mol^{-1}}, \tag{44}$$

on account of equations (15), (38) and (40). Moreover, in the present case in which most of the molecules are H_2, the explicit form of Λ is (see e.g. O'Dea et al. 1994 and Neufeld, Lepp & Melnick 1995)

$$\Lambda \simeq 3.8 \times 10^{-31} \left(\frac{T_\Delta}{\mathrm{K}} \right)^{2.9} \mathrm{erg\ s^{-1}\ H_2^{-1}}, \tag{45}$$

which – thanks to equation (41) – becomes

$$\Lambda \simeq 3.4 \times 10^{21} \Xi^{-2.9} \left(\frac{\mathrm{km}}{a_1} \right)^{4.35} \mathrm{erg\ s^{-1}\ H_2^{-1}} \tag{46}$$

(note that Λ is almost independent of M_{DC}). Now, from equations (44) and (46) it follows that the condition $\Lambda \gtrsim \Lambda_*$ implies

$$a_1 \lesssim 6 \times 10^{11} \Xi^{-0.7} \left(\frac{r_{\mathrm{m}}}{\mathrm{pc}} \right)^{0.35} \mathrm{km}. \tag{47}$$

Again it is difficult to figure out the relevance of condition (47) in general, but in the illustrative case of $M_{\mathrm{DC}} \simeq 10^5\,\mathrm{M}_\odot$ it turns out to be abundantly met for hard primordial binaries.

Thus we conclude that the energy given up by primordial binary brown dwarfs and temporarily acquired by molecular clouds is efficiently radiated away, so that the clouds are not dissolved by frictional hardening.

As a final comment, we stress that our estimate for T_Δ should be understood as an upper bound, since thermal conductivity has been neglected. In addition, the above analysis implicitly relies upon the assumption $T_\Delta < 10^4\,\mathrm{K}$, which ensures the survival of H_2. Actually, in spite of the fact that equation (42) implies that this may well not be the case, our conclusion remains nevertheless true; higher temperatures would lead to the depletion of H_2, which is correspondingly replaced by atomic and possibly ionized hydrogen. As is well known, in either case the resulting cooling rate would exceed the one for H_2, and so cooling would be even more efficient than estimated above (Böhringer & Hensler 1989).

8 CONCLUSIONS

We have shown that – within the considered model – the overwhelming majority of binary brown dwarfs are so hard today that their orbital radius is smaller than (roughly) one-half of the Einstein radius for microlensing towards the LMC, thereby making them unresolvable in the microlensing experiments performed so far. Still, we have seen that not-too-hard primordial binaries – whose initial orbital radius obeys equation (31) – have an orbital radius today which typically turns out to be about one order of magnitude smaller than the above-mentioned Einstein radius. We therefore argue that not-too-hard primordial binary brown dwarfs can be resolved in future microlensing experiments with a more accurate photometric observation,[9] the signature being small deviations from standard microlensing light curves (Dominik 1996). Note that such a procedure complements the detection strategy for binaries suggested by Gaudi & Gould (1997). Finally, we point out that the mechanism considered here can naturally account for an average MACHO mass somewhat larger than $\simeq 0.1\,M_\odot$.

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[9]See, e.g., the ongoing experiments by the GMAN and PLANET collaborations (Proceedings of the Second International Workshop on Gravitational Microlensing Surveys, Orsay, 1996).